Teaching the Art of Physics: Rationalizing and Formulating an Aesthetically-Oriented Physics Curriculum

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Abstract

This capstone is an attempt at creating a physics curriculum that capitalizes on profound questions of aesthetics that underlie basic principles. In Part I, I describe how aesthetics have contributed to my own love of math and science, how current curricula do and do not adequately address aesthetics, and how exactly scientists have recognized the beauty and aesthetics in their work. I conclude that the main sources of aesthetics in science, and what should constitute the backbone of an aesthetic curriculum, are abstract visualization, emergent characteristics like symmetry, and the contemporary scientific and intellectual context. In Part II, I discuss ways in which aesthetic math and science might be taught effectively and accessibly, and without derailing the normal curriculum. I propose that one answer may be visualizing and interpreting fine art as an analog for scientific theory. In Part III, I provide three examples of aesthetic physics lesson plans designed to be used in the Yale University Art Gallery.
Part 1: Introduction and Motivation

I generally consider my greatest skill to be an active awareness of the space around me—how it is stretched and squeezed, how things move through it, and how it is communicated. As an athlete, I credit this awareness for any talent I have in weaving between opponents or in sensing where a ball will be half a second from now; as a chronic doer-of-doodles, I thank it for the ability to read dynamic and intriguing spaces from even the most half-heartedly scribbled hatch marks. This sense of space is also the most basic tool with which I approach my coursework as a physics major, and the tool that helped physics become fun and intuitive for me in the first place. When I do my problem sets, my first step in analyzing a system is to stretch apart the components, rotate them as a single rigid body, then let them snap back into place. In electricity and magnetism, I think of the smooth curves of electric potentials and magnetic fields as rolling hills that I could run up or slide down. Only after making myself at home in the space of the problem do I draw a diagram and begin to work with any equations. And having already made sense of and mentally logged the different relationships in the system, the implications of those relationships and the correct logical progression to the problem’s solution always seem to follow more quickly.

In addition to its use in parsing complicated problems, building mental playgrounds gives me a unique way of interpreting—and in special cases, appreciating—characteristics of physical properties. In these special cases, it becomes the vehicle by which I can experience the properties in a more visceral way. Take, for example, the solution to Laplace’s equation, which describes the electric potential in a region of space free of electric charge: It turns out that the
solution,¹ the magnitude of the electric potential inside a two-dimensional boundary of given magnitudes, has no local minima or maxima (see figure 1). Or, in other words, it can be described as the surface that connects to all the edges using the smallest possible surface area. This does, of course, offer fertile ground for visualization. I could start by imagining an even drumhead and its flat solution, then bend the rim in my head and watch the solution surface stretch and twist to abide by the minimizing constraint. Or I could imagine myself the designer of a big, waterpark-style waterslide, and be comforted knowing that none of my patrons could be stranded in an eddy. But through these I can’t help but to recognize certain other characteristics and consider their significance.

In this respect, my reaction to the solution of Laplace’s equation has always been a state of being part affirmed and part surprised, alternating in proportion, and I’m not sure which came first. On the one hand, that such elegance and economy should manifest themselves in a fundamental characteristic of nature seemed obvious: a lack of electric charge seems to match the solution surface’s minimizing constraint. But on the other, the elegance seemed to impose an understanding of nature that I wasn’t sure I was ready to accept: why must nature be as smooth and economical as I want it to be? Is this a fundamental pattern of nature or just the necessary result of a second order homogeneous partial differential equation? What is certain is that sometimes it is possible for me to feel physics as much as I can understand it, and that

¹ “It turns out” is a loaded phrase. It usually means some complication is being swept under the rug and that an instructor is asking you to take their word in lieu of a rigorous proof. The simplification it manifests is often surprising, and itself is often the origin of the types of “aesthetic” moments I describe later.
through these “aesthetic moments” and because of them, I engage with the subject material more actively, and am more driven to pursue the questions they raise.²

It seems to me that this type of experience should be shared with all students, if possible. In learning about Laplace’s equation, it is of course important to know how and why we arrived at it, that the equation reads $\nabla^2 V = 0,$ that the upside-down triangle squared times $V$ means the divergence of the gradient of the electric potential $V,$ that divergence and gradient have their own mathematical definitions, and that to derive the equation and its useful properties in the three dimensions we actually live in (rather than the two-dimensional world where magnitudes can be imagined as heights of waterslides), it takes some technical, not emotional, calculations. But looking back, it is also significant that what sticks with me from my sophomore year Electricity & Magnetism class are not the equations or calculations, but the qualitative understanding and David J. Griffith’s Introduction to Electrodynamics textbook that it came in. The aesthetic moment not only captivated me during the class and stuck with me after, but also helped me form and hold onto a deeper understanding of the material. To mobilize this aesthetic moment as a means of teaching is the goal of this project.

The sections below will be ordered as follows: First I will consider the Science Technology Engineering Art and Math (STEAM) movement and how its integration of art actually differs from an aesthetic approach in the way I propose. Then, I will try to develop a more robust understanding of the aesthetic in math and science by supplementing my own

² Ernst Fischer, Beauty and the beast: The aesthetic moment in science, (New York: Plenum, 1999). I like the terminology “aesthetic moment” because more so than just “aesthetics” in science or “beauty” in science, it really emphasizes that it is an experience—and thus retains some aspect of the indescribable that the other terms can make you forget about.
experiences with the accounts of some of science’s biggest names, as well as secondary commentaries that have already begun to deconstruct them. The purpose of this section is to arrive at some explanation of how the aesthetics could be implemented in the classroom. Then, I discuss ways in which aesthetic math and science might be taught effectively and accessibly, and without derailing the normal curriculum. Finally, I provide three example lesson plans that implement these ideas.

STEAM Education and Current Practices

This project comes at a time when science and math education is changing rapidly in response to underperformance and underrepresentation. Many reformers, including President Obama, have called this perceived crisis a modern day “Sputnik moment,” referencing the United States’ intense prioritization of technological preparation and competition at the start of the Cold War. Many solutions propose organizational and extracurricular changes, such as establishing student-centered mentorship programs, incorporating comprehensive multimedia instructional supports, and modifying assessments to demonstrate growth rather than measure knowledge, just to name a few. This project, however, focuses specifically at the level of the

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classroom. While broader reforms are necessary and have been shown to be effective, they are ultimately beyond the scope of this capstone.

STEAM education comes closest to the program of aesthetics that I propose in this capstone. Having risen to prominence only recently, STEAM in theory uses science, technology, engineering, arts, and mathematics “as access points for guiding student inquiry, dialogue, and critical thinking.”STEAM is itself a response to the huge emphasis on STEM and the push to increase American representation in STEM careers. Educators of the humanities argue that STEAM better represented the “vitally important” and neglected “symbiosis between the arts and sciences.” Along with industry representatives, they also argue that innovation, the true aim of American participation in STEM, is born from creative, artistic minds. In theory, these goals are actually similar to mine. In practice, however, I find STEAM curricula often leave a lot to be desired.

For example, a partnership between an art gallery and natural history museum in Utah provides a rationale similar to mine: In their opinion and according to their research, “the arts animate education because they are inherently experiential and because of their potential to develop creative and critical thinking skills in students. These same skills are valued in science, technology, engineering, and math (STEM) education, but the arts have not been consistently

7 ibid.
included in STEM lessons.”

The museums’ implementation, however, looks very different from what I would propose. In one of the iterations of the museum partnership, programming included learning about species of fish, then using casts of the fish to make prints. I do not doubt that this was, in fact, fun and informative, but it does leave the connections between art and science somewhat superficial. It seems to suggest only that an object can be both beautiful and described in biological terms. It falls short of encouraging students to consider the beauty of the biology itself.

Two characteristics of the museum program represent themes in current STEAM education that this project seeks to improve. First is its focus on biology, which is understandable, given it is a Natural History Museum, but which also serves as a convenient example of the rarity with which even STEAM addresses math and physics in extracurricular and classroom settings. The “M” is often the most underutilized letter in STEAM elementary and middle school course catalogs, and in the last eighty issues of The Journal of Aesthetic Education, only one article has discussed the connections between math and art education.

The second is that many STEAM curricula, including that from the Utah natural history museum, aim to enhance STEAM curriculum by using art to illustrate and punctuate, rather than combining the study of art and the study of science as mutually complementary disciplines. To the extent that aesthetically-oriented math and physics curricula currently do

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exist, they do not address the aesthetic moment in a meaningful way. Any semblance of aesthetic considerations is amalgamated into the “A” for art, and what is promoted as integrated math and art curricula often connects the two in superficial and purely instrumental ways. “Proportional People: Math and Art,” for example, uses people of different sizes to describe proportional relationships. The connection between art and math here is purely descriptive and so trivial that is unlikely to inspire wonder or any further inquiry. In the same way, STEAM’s emphasis on application-based learning often clings too tightly to practicality. In contrast to these approaches, I argue that it would be even more valuable to unite art and science in a way that emphasizes their more philosophical interpretations and implications.12

The goal of this project is not to correct existing STEAM curricula, but to offer another perspective, whose goals might be less bound by a demand for demonstrable and measurable learning outcomes. The added value of aesthetic approaches would appear in conversations students wouldn’t have had otherwise, not in test results. In fact, more so than as a response to any current educational initiatives, this project was conceived of as harnessing the same excitement that can be found on reddit boards discussing interpretations of the movie *Interstellar*. Students would undoubtedly benefit from that zeal, and the very existence (not to mention commercial success) of contemporary sci-fi movies and pop-science literature genres suggest at least some potential to captivate students in a setting of formal education. While some blockbuster science-fiction movies are more educational than others, the abundance of

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12 This is not to say that “Proportional People” is necessarily a wasted curriculum, though. It involves mathematical and visual perception, both of which are understood to be important to artistic and scientific development. I also suspect it connects to such skills as the spatial awareness I described above, and in this manifestation, it is a useful tool. But ultimately, only a tool.
pop-science memoirs actually reflect a growing use of storytelling and historical vignettes as a means of capturing the scientific imagination. The authors themselves make their own intentions clear: scientific concepts should at least in part be taught through a pedagogy of that emphasizes the wonder felt by scientists who discovered and use them. Ian Glynn, a biology professor at the University of Cambridge and author of *Elegance in Science: The Beauty of Simplicity*, considers it a pity that young students do not learn the formula for the area of a circle in the same way Archimedes elegantly proved it; disregarding this approach means eschewing a potentially formative scientific experience. Edward Frenkel, a UC Berkeley mathematician and author of the *New York Times* bestseller *Love and Math*, elaborates on this disappointment:

> What if in school you had to take an ‘art class’ in which you were taught only how to paint a fence? What if you were never shown the paintings of Leonardo da Vinci and Picasso? Would that make you appreciate art? Would that make you want to learn more about it? I doubt it... but this is how math is taught, and so in the eyes of most of us it becomes the equivalent of watching paint dry.

Instead of watching paint and any desire for a career in science dry to the point of crumbling, I agree that we should be stimulating students to enable them “to appreciate the excellence of a work, and to induce [them] to an attempt to produce good things [themselves].”

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Drawing insight from the reflections of professional scientists such as these, in the next section I will further explore and build on their observations.

**The Aesthetics in Science**

In order to write a curriculum for an aesthetic math or physics class, it’s important to understand what exactly it means for math or science to have an aesthetic component, or to “be beautiful.” For this I turn to the scientists whose discoveries have already captured the world’s imagination.

Before we understand precisely what aesthetics in science are, though, a couple things may be said. First, we can definitively say that aesthetics is important to a lot of scientists; conversations about aesthetics are ubiquitous in science. To some, the beauty to be uncovered in nature is science’s main motivator. Nobel Laureate Subrahmanyan Chandrasekhar, for one, argues that the divide between the arts and sciences is nonsensical. After all, he says, “in the arts as in the sciences the quest is after the same elusive quality: beauty.” \(^{17}\) He references Henri Poincaré, who, in his own reflections, remarks that this beauty is more than incidental, it is the precise reason science is studied at all. \(^{18}\) Sabine Hossenfelder, an author and theoretical physicist at the Frankfurt Institute of Advanced Study, agrees. Even though she disapproves of beauty’s influence on some of her peers’ actual research, she never tries to deny its existence. She even ventures to question “whether anyone would care about the laws of nature if they weren’t beautiful.” \(^{19}\)

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\(^{17}\) Subrahmanyan Chandrasekhar, *Truth and beauty*, 52.

\(^{18}\) *ibid*, 54-60.

Second, in shaping actual scientific practice, to the dismay of critics like Hossenfelder, the pursuit of beauty might be even more apparent. Different scientists assign different weights to beauty. Chandrasekhar simply considers “beauty the splendor of truth.” But this relationship is often inverted, and truth becomes a function of beauty. Herman Weyl describes his role as a mathematician-physicist-philosopher: “My work always tried to unite the true with the beautiful; but when I had to choose one or the other, I usually choose the beautiful.” Hossenfelder points to many more examples of the pursuit of beauty leading scientists astray, and it is easy to condemn this subversion of logic to beauty. At the same time though, this subversion can still be found behind some of the most monumental discoveries in science. In astronomy, for example, Copernicus’s rationale for placing the sun, rather than the earth, at the center of the galaxy was driven by an aesthetic hierarchy. He describes the sun as “enthrone[d]” in the middle of all, and that it is from “this most beautiful temple” that the sun should shine. In divining the laws of planetary motion, Kepler similarly defends the sun’s place in the center on the grounds of its status as the primary beauty of the world. The scientific value of prioritizing beauty is certainly dubious, but I maintain that this dilemma isn’t only harmless in small doses, but also that the contention might even spark the debates that could motivate students to pursue science.

Hossenfelder objects to any deference to beauty as an arbiter of truth. She does admit, however, the ability of these aesthetic considerations to captivate audiences. I believe she would be in favor of this curriculum.

20 Chandrasekhar, Truth and Beauty, 54.
21 Ibid, 65.
22 Astronomy is the discipline today that Chandrasekhar considers the most directly connected with antiquity—a time when many intellectuals were polymaths and the distinction between science, philosophy, and the arts was much less discernible.
23 Fischer, Beauty and the beast, 1.
24 Ibid.
Let’s return to our search for a definition. What does the beauty in science actually mean? It turns out that there isn’t a single comprehensive definition, but as a whole, three categories seem able to account for most testimonies. The first category is the extent to which a scientific concept lends itself to visualization. In philosopher of science James McAllister’s 2002 review of recent work in aesthetics, he first reaffirms the influence of aesthetics in science, and he puts particular emphasis on its importance for theoretical physicists and mathematicians. McAllister argues that in the past few decades the major object of aesthetic consideration has been scientific images. Better computing technology has allowed for more flexible animation and more precise computation, making for images that can help reason through challenging problems or reveal degrees of organization impossible to see with the naked eye. Famous examples are the maps of the Four Color Theorem (see figure 2), and the fractals of the Mandelbrot Set (see figure 3). Michelle Emmer concurs, and credits recent use of computers and graphics in proving theories with the emergence of a new “visual mathematics.” Mathematicians in these new areas, says Emmer, seem to be naturally drawn to the beauty or “the aesthetics in the new forms discovered in their work.” Aesthetic images can be either concrete, like a computer-generated depiction of the Mandelbrot Set or photographs of observable phenomena, or abstract, like a scientist’s artistic representation of the unobservable. Most modern-day writers, according to McAllister, consider scientific images of both kinds to be inseparable from their content. In other words, what makes them beautiful is

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essentially that they “look cool,” and what makes them aesthetically intriguing for scientists is that their beauty belies less-obviously beautiful theory.

Before modern computing software, though, scientists still saw beauty in science. Without the visual aids to steal the spotlight, though, it was the theory itself that was considered beautiful. Moreover, theories that are declared as beautiful vary across scientists and across time periods, but the words used to describe them often stayed the same.\(^{26}\) If the first category of aesthetics in science is in a concept’s ability to be visualized, then the second category is the presence of certain “aesthetic” characteristics; theories or phenomena that are called beautiful often have specific quantitative or qualitative features like “unity” or “harmony”. McAllister reports that in a broad stroke, the aesthetic in the “mathematical sciences,” namely math and physics, can be summarized as “classical” and “formalist,” in which “unity, economy, symmetry, consistency, balance, harmony, order, and the like” are valued.\(^{27}\) Michele Emmer concludes that two main connections between pervasive theories are the “simplicity and clarity of definitions and statements making them beautiful” and “words that seem obvious but in fact conceal a profound truth.”\(^{28}\) Combined, they suggest scientists have considered the beauty of math and science to rest both in their formal elements and in the extent to which their empirical natures disguise very existential implications.

These first two categories neatly account for Chandrasekhar’s and Glynn’s reflections, and also my own mode of appreciating Laplace’s equation, but only because they remain rather


\(^{27}\) McAllister, “Recent Work on Aesthetics of Science,” 9.

\(^{28}\) Emmer “Aesthetics and Mathematics,” 255.
general and inclusive. This might be concerning in the pursuit of making a legible curriculum out of them. To be fair, however, they are only reflections of the scientists’ own vagueness.

Consider, for example, the idea of harmony, which is one of the most cited aesthetic qualities among both scientists and philosophers of science. Werner Heisenberg defines beauty as “the proper conformity of the parts to one another and to the whole,” and Henri Poincare finds beauty in the “harmonic classification of parts” and the extent to which new theories “introduced harmony in what was before chaos.” Chandrasekhar also admires the universality of the basic laws and describes scientific values as the “continuing and increasing recognition of uniformity of nature.” And to Scottish philosopher Francis Hutcheson beauty can be found in the persistence of uniformity within increasing variety. Although harmony seems to be well understood as an aesthetic characteristic, it is never formally defined or elaborated. The same can be said for abstract appeals to simplicity, order, et cetera.

Maybe being indescribable is an inherent quality of beauty, but this ambiguity also raises the concern that scientists are just using words arbitrarily—that one person’s harmony might be another person’s simplicity, and that each person’s “beauty” could actually be completely different. McAllister concedes that this objection is probably warranted in some cases, but he errs on the side of trust. Invoking the experiential nature of aesthetics in science, it is, in his eyes, tolerable to accept declarations of beauty at face value, not only according to the “the principle of charity of interpretation,” but also because whatever the terminology

29 Ernst Fischer, Beauty and the beast: The aesthetic moment in science, (New York, Plenum, 1999), 12.
30 Chandrasekhar, Truth and beauty, 52-60.
31 Ibid, 4.
might be, each scientist’s individual aesthetic “canon” the experience of its realization is necessarily legitimate.\(^{33}\)

McAllister’s own thesis in part deals with any ambiguity. He believes that the recognition of the Beautiful in science is not so much a self-standing, spontaneous expression, but rather a reflection of the norm accepted by the contemporary scientific community. This norm is shaped by confirmed and “fashionable” data and results.\(^{34}\) And here we come to the third general category: beauty must have a social dimension. In order for science to be beautiful, it must be acknowledged as such by the collective members of the scientific community. An example McAllister gives is the disdain with which some scientists at the turn of the twentieth century referred to the “ugliness” of quantum mechanics, contrasted with reverence of more recent scientists who declare the beauty of the same exact thing. The only difference, he points out, is that more recent scientists don’t have to remain skeptical of quantum mechanics because, by now, it has been accepted and confirmed experimentally many-fold. His theory implies that scientists normally treat beauty and truth in the “correct” way—the former possible if and only if the latter occurs first, as would satisfy Hossenfelder—but we have already seen that this is not always the case. With regard to a variation in aesthetic appreciation, he still leaves some room for personal canons but also for scientific revolutions, like the quantum revolution. A scientific revolution, according to McAllister, therefore, only occurs through those movements that fundamentally shift the aesthetic outlook of the scientific community. Formal qualities like symmetry and simplicity he dismisses as simply


\(^{34}\) Ibid.
projections of subjective criteria onto concepts that evoke aesthetic responses for other reasons.\textsuperscript{35}

Kant, Schiller, and Hegel, among others, have written extensively on aesthetics and the sciences, but to treat them fully would require another paper. For the purposes of this project, the three elements of beauty that I have discussed provide a solid foundation for thinking about how to design a mathematics curriculum that is attentive to its aesthetic dimensions. To review, these elements are (1) Visualization, (2) Aesthetic Characteristics, and (3) Collective Scientific Norms. The three overlap (Aesthetic Characteristics especially spills over into the other two) but that’s okay—theys form a basis of aesthetic accounts of science, and constitute a good road into aesthetics for the students this capstone is designed for.

As we move on to think about how to design a mathematics and physics curriculum that incorporates aesthetics, it’s also important to consider the typical means of discovering beauty. That is, when are scientists most predisposed to noticing beauty? It turns out that, in about equal terms, beauty lies in scientific processes and in the moments of discovery. By scientific processes, I do not mean the Scientific Method, but the implementation of specific mechanisms, like complex math in an elegant derivation. Nobel Laureate Eugene Wigner contemplates math’s “miracle of appropriateness for the formulation of the laws of physics,” for example.\textsuperscript{36} Moments of discovery are more straightforward. It can could be a new discovery or a confirmation of an old theory: Chandrasekhar, for example, “shuddered before the beautiful” when it was discovered that the exact solution of General Relativity exactly

\textsuperscript{35} McAllister, Beauty and revolution in science, 33.
represented massive black holes. My own experience of aesthetic beauty was from a result, what McAllister describes as an agreement of a scientific implication and a particular scientist’s metaphysical worldview. I like to view nature as not wasteful; therefore, the economy of the solution to Laplace’s equation really excited me. This temporal consideration is important for the ultimate curriculum. While the three general categories of beauty in science will dictate the content of the curriculum, understanding what delivers scientists to these aesthetic moments might inform the methods of conveying that content. Understanding that aesthetic moments occur in both results and processes helps ensure a lesson that faithfully recreates scientists’ experiences.

Finally, let it not be understated how meaningful aesthetic experiences can be. Many of the scientists who write about aesthetics are so moved because—whatever their aesthetic experience—their models and formulas seemed to be more than the sum of their parts: they meant more than the predictive power that they could generate. This returns to the relationship between empirical truth and existential implications. It seems that science, and especially pure math and theoretical physics, has a capacity to induce an existential reward that for scientists is nigh-religious. It is no wonder science and religion were inseparable for so long. Wigner sees miracles, and Chandrasekhar’s writing also suggests a transcendence from empirical, scientific truth to a higher, capital-t, Truth:

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38 To fully understand the role of scientific processes in aesthetics, it would be necessary to introduce the concept of creativity. Although it is certainly relevant, a comprehensive discussion of creativity lies outside the scope of this paper. For more on creativity and aesthetics in science, see: Miller, Arthur I. *Insights of genius: Imagery and creativity in science and art*. Springer Science & Business Media, 2012.
The pursuit of science has often been compared to the scaling of mountains, high and not so high. But who among us can hope, even in imagination, to scale Everest and reach its summit when the sky is blue and the air is still, and in the stillness of the air survey the entire Himalayan range in the dazzling white of the snow stretching to infinity? None of us can hope for a comparable vision of nature and of the universe around us. But there is nothing mean or lowly in standing in the valley below and awaiting the sun to rise over Kinchinjunga.\textsuperscript{39}

The universe is beautiful, and getting to know it is too. This is what Chandrasekhar sees in science, and these feelings consistently inspire and surprise him. Students should have the opportunity to feel the same way.

\textsuperscript{39} ibid, 26-27
Part I Figures

Figures 1(a), 1(b)

Figures 1(a) and 1(b) depict a solution to a Laplacian with boundary the boundary condition \( z=0 \) on all sides except for one. On the last side, the value of \( z \) is constrained to equal the value of a sine wave. Notice that the only minima or maxima are at the boundaries. (Images made with MATLAB_R2020a.)

Figures 2(a), 2(b)

Figures 2(a) and 2(b) display the result of the Four Color Theorem, that any map of contiguous shapes can be colored with no same-colored shapes touching each other using only four or fewer colors. After more than a hundred years of false proofs, researchers in the 1960s used computers to manually check all possible cases and prove that four colors was, in fact, all that you needed. (Images retrieved on May 1, 2020 from Wikipedia, https://simple.wikipedia.org/wiki/Four_color_theorem)
The Mandelbrot Set concerns the complex-valued function $f_c(z) = z^2 + c$. In the images above, the black regions represent values of $c$ for which the function does not diverge when iterated from $z = 0$ and the dark blue region represents values for which it does diverge. Other colors mean that there is small-scale complex behavior and more magnification is needed to see it.

The images above, which zoom in progressively from top left, to top right, to bottom left, to bottom right, show the amazing patterns and self-similarity that these regions turn out to possess.

Part 2: What Kind of Curriculum

Art and the Classroom

This capstone attempts to teach an aesthetic consideration of math and physics using fine art. Using art is promising for two reasons. First, the whole notion of aesthetic math and science appears to be extremely open-ended—as noted above, there are only some overlaps in scientists’ definitions or chosen qualities that differentiate aesthetic math and science from unaesthetic. So it might be hard to imagine how to recreate aesthetic experiences in the classroom, considering both the time constraint and the task of abstracting already abstract material. What may be possible, though, is focusing on the similar responses to aesthetic science and the fine art. Artists and scientists alike have described their parallel philosophies of art and science. Art critic Clive Bell relates the impractical feelings of beauty for beauty’s sake experienced when looking at a piece of art and rapt in a math problem.\(^\text{40}\) Theoretical physicist Werner Heisenberg, speaking at an art convention, recounts the “helplessness when faced with the question of what to do about the bewildering phenomena” which has “shaped the face of both disciplines and both periods, different as they are, in a similar manner.”\(^\text{41}\) Heisenberg recognized that art and science are, at their cores, exercises in making sense of the world. In this way, art might provide a sort of backdoor into the precise type of discussion I hope to foster.

Perhaps more importantly, it seems that it is actually possible to use art as a backdoor into a discussion of aesthetics. Beautiful art has been shown to elicit similar emotions as

beautiful science in scientists and laymen alike. That is, art does not just recreate the same feelings as aesthetic math and physics—it does so in an accessible way which might be more familiar to students. Neuroscientists have found that formulas and works of art activate the same area of the brain in scientists,\(^{42}\) and a recent study of non-scientists has shown that they, too, are able to associate formulas and works of art with the same feelings at a statistically meaningful rate.\(^{43}\) It turns out Hutcheson’s concept of the mathematician’s “sixth sense,” which supposedly lets the mathematician alone appreciate mathematical beauty, is a fallacy.\(^{44}\) The appreciation can be raised in anyone, including the students whom this capstone aims to serve.

There are, of course, other approaches. One is to highlight clever and unintuitive proofs to liven math and physics content. Ian Glynn, for one, thinks early math lessons could benefit from using clever proofs like Archimedes’ of the area of a circle, for no other reason than it makes you pause and think “Wow that’s cool!” These types of problems tend more toward abstract problem-solving than traditional and linear math pedagogy, they are abundant and thought-provoking, and they can be used to generate a lot of interest in a topic. The popular YouTube channel Numberphile does just that. It highlights unique problems in videos such as “Infinity is bigger than you think” and “Pi is beautiful,” its contributing mathematicians balancing comprehensive explanations and genuine wonder in their dialogue with the channel’s more than three million subscribers. “Infinity is bigger than you think” and “Pi is beautiful” together have eight million views, and others individually have more than fifteen million—


\(^{44}\) Michele Emmer, “Aesthetics and Mathematics,” 245.
educators would certainly be hard pressed to find traditional online lessons with such eager students.45 Another approach is integrating math and science with the history of math and science in a way that focuses on the mathematician or scientist as a person. This represents a sort of evolution of the math-themed stories sometimes used in early childhood education. The motivation is that contextualizing an abstract topic in the figures and stories that led to their discovery would both reduce subject anxiety and increase interest.46

Both of these, though, are challenging to craft entire lessons around. Most obviously, while counterintuitive proofs are abundant, they are not omnipresent, and there isn’t an example for every topic at the right level of complexity. The same goes for interesting historical vignettes. So, in order to effectively include them, a piecemeal approach would be necessary. But sticking to a loose commitment like to include these asides during a normal class is difficult for the reason that haunts so many teachers: time. There is just not enough time in the school day or year to cover both all the required curriculum and any meaningful supplemental content, and no matter the philosophy, the choice between scattered in-depth supplements and a sound fundamental knowledge is probably clear.47

This suggests that any realistic intervention might not be able to take place in the math or physics class itself. A solution Michael Fried offers for the time constraint is “radical separation,” in which the actual math content and the humanistic side are given their own, separate class time.48 This also appeases Sabine Hossenfelder’s objection that an aesthetic

47 Fried, "Can mathematics education and history of mathematics coexist?," 391-408.
48 ibid, 391-408.
math and science is often at odds with good math and science. Rather than presenting both approaches as married in the same course, separating them retains the connection—and hopefully the motivating effects—without necessarily asserting their agreement.

This line of reasoning gives opens the door for another big reason to implement art: the Yale University Art Gallery (YUAG). The museum receives hundreds of visiting K-12 classes every year, and offers guided tours that use items in the collections to spark discussions about topics in history, geography, and social studies that the students have been learning about in school. Needless to say, the quality of the collections and the benefits of viewing them in person are reason enough to tie the YUAG into this project by any means possible, but the YUAG’s model for class visits also presents an equally suitable balance between meaningful exposure to supplemental content and minimal disruption of the fundamental material.

Students only miss one day of normal class, and it seems harder to trivialize the relevance of art when you see it in person. I also think the art in the YUAG happens to have a lot of unintuitive potential in discussing the aesthetics in math and physics especially.

In the Gallery

The end product of this capstone are modules that will point out specific pieces in the gallery and pose discussion questions that are tailored toward high school students and that focus on the visualization physics principles, the implications of “aesthetic characteristics,” and the significance of the intellectual context of the discovery. They will be based on similar resources that already exist at the Yale University Art Gallery and the Metropolitan Museum of
Art. Each has its own style, but they can be broadly understood as being either mostly “skills based” or mostly “content based.”

At the YUAG, exercises and guided tours are skills-based: centered around observation, open-ended questions, and discussion. For differently themed tours, guides might shape discussion by “framing” works of art in different ways, but they still let the works themselves act as the “primary vehicles through which... larger connections and contextualizations can be made.”49 The physical materials include handouts for students, which might be objective-versus-subjective observation prompts and creative activities, such as the word association games. Other YUAG activities incorporate sketching, felt shapes, and disposable cameras.50

The Met’s guides are mostly adapted from teachers who have created them as lesson plans for their own class and later reach out to the museum to share. This means they tend to be more tailored to concrete learning goals and national learning standards. In “The Making of a Persian Royal Manuscript,” for example, goals include being able to identify “key events and figures presented in the Persian national epic.”51 The lesson plan presents students with background readings and videos, and it points teachers to specific items in the collection, and explains how they fit into the lesson plan.

My modules combine strategies from the Met’s and YUAG’s lesson plans, in that each of mine will include substantial content-specific background information, but will also try to keep the time in the gallery very open-ended by incorporating skills-based games and activities. All of

50 Rachel Thompson, email message to author, February 5, 2020.
the modules, like all of the Met’s and the YUAG’s, will consist of multiple worksheet handouts for the students and background information for teachers, and they will include clearly outlined learning goals, suggested pieces to use and time allocations to abide by. Several activities, such as the “What is the Most” word game, will be borrowed directly from the YUAG.
Part 3: The Curriculum

Below I present one module for each of the three themes I found to describe an aesthetic science, which were 1) the extent to which concepts lend themselves to visualization, 2) the presence of certain qualitative or quantitative characteristics deemed “aesthetic”, and 3) a community-wide shift in aesthetic values in response to “fashionable” theory and experiments. Each plan is meant to be used in conjunction with collections in the Yale University Art Gallery. The lesson for the first category, called “Vectors and Space,” focuses on the visual component of vectors. In it, students are encouraged to use paintings to visualize the ideas of vectors, linear independence, and dimension. The lesson for the second category, called “Symmetry and Emergence,” challenges students to question the nature of symmetry in different works of art, and especially whether any symmetry was strictly encoded into the piece’s creation, or if it simply emerges out of more chaotic foundations. The last lesson is called “The Quantum, Continuity, and Abstraction.” In it, students use progressively fragmented paintings to visualize basic principles of quantum theory, as well as observing how a system of aesthetics might progressively develop in both art and science.

Each lesson has materials for the teacher, which include logistics and background information on the topic and suggested paintings, as well as materials for the students, which include the background on the topic and guiding questions and activities. Although there is significant background information, which would ideally be reviewed before visiting the gallery, the guiding questions remain observation-based, and very open-ended. Similarly, although some experience in math and physics is helpful, the background is meant to be comprehensive enough, and the questions open-ended enough, that any student can participate.
Vectors and Space

Lesson Overview

In this lesson, students will use selected paintings to help them visualize vectors and use them to describe space. For each painting, students will consider the following questions:

1. What do you see? Does anything seem like it might be a vector? How do you know? Is it big or small? What direction does it point?

2. If you can find any vectors, do they seem to interact with each other? How do their sizes relate? Are they parallel? Perpendicular? Somewhere in between? How many dimensions do they generate?

3. Do these relationships affect the painting as a whole? If so, how? Are they reminiscent of any other scenes or ideas?

Materials Overview

For this lesson, each student will need a writing utensil and paper. In addition to worksheets for each student, the instructor might find it helpful to carry extra sheets of blank paper if any student needs more room to write or draw. The lesson specifies three paintings in the YUAG’s Modern and Contemporary Art collection, but others may be used as well.

The paintings selected for this lesson are Pan North XI (Al Held, ca. 1987), Brass Band (John Covert, 1919), and Brooklyn Bridge (Joseph Stella, 1919-20). For this topic, however, a lot of paintings could make very good examples. The teacher or guide may select others as desired. Paintings that use line, implied line, or perspective might be most easily applicable.

Suggested Itinerary. Total Time: 1 hour 15 minutes

<table>
<thead>
<tr>
<th>Time</th>
<th>Location</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>–</td>
<td>YUAG Lobby</td>
<td>○ Welcome and Introduction&lt;br&gt;○ Lead group to Modern and Contemporary Art collection</td>
</tr>
<tr>
<td>15 minutes</td>
<td>Pan North XI</td>
<td>○ Introduce the concept of vectors, emphasizing</td>
</tr>
</tbody>
</table>
### Background on Vectors:

#### What is a vector?

Vectors are quantities that have magnitude and direction. They are all-important in physics because they describe quantities that exist in the physical, three-dimensional world. One such quantity is displacement, which is defined as a change in something’s position. Like all vector quantities, it needs direction to have physical meaning: if you move, you have to move in some direction. Moving in no direction is equivalent to not moving at all.

#### How do vectors define space?

Like displacement, all vectors involve a change in some quantity, and as such are depicted as arrows connecting two points. To combine vectors, you pin the tail of the second vector to the

<table>
<thead>
<tr>
<th>Time</th>
<th>Collection</th>
<th>Activity</th>
</tr>
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<tbody>
<tr>
<td>25 minutes</td>
<td><em>Brass Band</em></td>
<td>○ Introduce painting (Title, Artist, Year)</td>
</tr>
<tr>
<td></td>
<td>– American Paintings and Sculpture Collection</td>
<td>→ Observation #2 Questions</td>
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<td></td>
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<td>→ After 5 minutes, discuss student responses for another 5 minutes.</td>
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<td></td>
<td></td>
<td>○ Transition to “What is the Most?” activity.</td>
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<td></td>
<td></td>
<td>→ Tell students to spread out and to use all the paintings in the room.</td>
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<tr>
<td></td>
<td></td>
<td>→ After 10 minutes, gather and discuss answers.</td>
</tr>
<tr>
<td>35 minutes</td>
<td><em>Brooklyn Bridge</em></td>
<td>○ Introduce painting (Artist, Year–don’t mention title!)</td>
</tr>
<tr>
<td></td>
<td>– American Paintings and Sculpture Collection</td>
<td>→ Observation #3 Questions</td>
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<td>→ After 10 minutes, discuss student responses for an additional 5 minutes.</td>
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<tr>
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<td></td>
<td>○ Drawing activity</td>
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<td>→ After 20 minutes, return to the group and discuss.</td>
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<td></td>
<td>○ Conclude, have students share anything they’ve learned.</td>
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</table>

| 25 minutes | *Pan North XI* (Title, Artist, Year)            | Observation #1 Questions                                                 |
| 25 minutes |                                                 | After 5 minutes, discuss student responses for another 5 minutes.         |

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- Generating sets and dimension.
  - If students have not read background and overarching questions, give them time to do so.
  - Introduce *Pan North XI* (Title, Artist, Year)
  - Observation #1 Questions
  - After 5 minutes, discuss student responses for another 5 minutes.
tip of the first. The resultant vector is the one made by connecting the start of the first vector
directly to the end of the second. If the vectors are parallel, this is the same as taking just one
step forward instead of three forward then two back. If the vectors are perpendicular, combining
them is like taking five steps at an angle, rather than three forward then four sideways.

It’s also possible to stretch or squeeze a vector. This gives you more options when trying
to compose a new one. Instead of taking three steps forward and two back, for example, you
could also just take $3 \cdot \frac{1}{3}$ sized steps forward and reach the same destination. In one
dimension, it is actually possible to create any vector just from stretching or shrinking any other.
But the key point is that when you stretch a vector, you can’t pivot it. So in two dimensions, you
need at least two non-parallel, or linearly independent, vectors to make any other. It’s like the
difference between a number line and an x-y graph. On the number line, every point can be
described by a distance from zero. On an x-y graph you need two distances, one in the x-direction
and one in the y-direction. This is the concept of a basis: a set of the minimum number of vectors
needed to compose any other vector in space. A basis generates the space. The dimension of a
space is the number of vectors in one of its bases. Then in order to describe any vector in three
dimensions, you need a third direction, the z-direction. And so on.

\[ \text{A} \rightarrow \text{B} \quad \text{A} + \text{B} \]

The leftmost picture shows two parallel vectors combining. Since the two of them are not linearly
independent, they only generate one dimension. The middle picture shows two linearly
independent vectors combining. They span two dimensions. The rightmost picture shows three
linearly independent vectors combining. They generate a three-dimensional space.

**How is this different from a coordinate plane?**

The essence of what we’ll do is not so different from constructing coordinate graphs in
painting. The most convenient bases consist of vectors that are perpendicular to each other, like
the x, y, and z-axes. But it is still possible to span a given space with skewed axes; the recipe for
a given vector will just change a little if it is to be in terms of the new axes. (This exact concept is
often used in special relativity, in graphs called Minkowski diagrams, to describe frames of
reference of objects travelling at different speeds.)

Skewed axes are interesting to look at because they might define two dimensions in a
suboptimal way, but they also happen to be the way we often depict more dimensions. In these
cases, we place additional axes skew to the x and y-axes and assume that they are all
perpendicular, even though they don’t seem so on paper. Ultimately, it is up to the viewer to
interpret the “new” axis as depth, rather than as just a diagonal line. Perceiving a third dimension
obviously comes quite naturally a lot of the time. But in more abstract works, it can be an exciting
exercise to judge the dimensions at play, and to grapple with the limits of the medium and of our
own capacity to perceive dimension. Adding to this exercise is the concept of perspective, which
is the effect that makes lines stretching into the distance seem to converge at a distant point.
Perspective also makes you consider which lines are really parallel and which are not. The activities in this lesson aim to encourage these considerations, using the vocabulary and concepts of vectors.

Finally, a point of clarification: if we consider the space in paintings, it is important to understand that physical objects themselves are not vectors. The edge of a block, for example, might be oriented along the x-direction vector, or it might make viewers think of certain vectors, but the edge is not, strictly speaking, a vector itself.

Background on **Pan North XI**

Al Held finished *Pan North XI* in 1987. It is an example of his period of Geometric Abstraction, which was in part a response to his previous abstract expressionist periods, whose paintings he felt were too flat. Keeping with his desire to add depth, he makes space in this painting very clearly legible. He very clearly established a vertical direction—almost every shape is rectangular and has one edge that is straight up and down. All of these edges suggest a y-direction vector that points straight up and down (remember: the edges themselves are not vectors because they are just “material” objects. The only direction they have is their orientation along a direction vector.) At the ends of these vertical edges are horizontal edges. Some are perpendicular, such as on the dark, plenum space-reminiscent shape in the upper third of the painting. This shape also has several points at which a third line comes into the same vertex at a near forty-five degree angle, which makes it very tempting to assume that this third line is meant to represent a third perpendicular line in the third dimension, and to adopt this set of three directions as our default coordinate axes.

This paradigm (or whatever x-y-z axis is chosen first) lets you compare lines that don’t directly fit, such as the floating rectangular plates in the bottom half of the painting. Their tilt makes the space feel less restrictive; the edges don’t have to fall into the grooves of our established axes, but are free to move about. By successfully spanning three dimensions in a different way, they might make viewers realize that the initial axis selection is arbitrary. Similarly, the edges that reach back into the painting with different lengths and towards different vanishing points contribute to the vastness of the space in the painting’s background.

It is also interesting to consider how color augments the weave of direction vectors. This painting includes a lot of vertices at which three planes meet each other, and each having a different color helps to differentiate them and to elucidate which planes are broken up by which edges. Imagining what this painting would look like in lines only forces viewers to consider the extent to which lines, by themselves, are able to describe spaces, and possibly the extent to which they, as viewers, are even able to read space from lines.

Background on **Brass Band**

*Brass Band* is interesting to consider because it lacks many of the techniques that help viewers make sense of *Pan North XI*. First, whereas the long straight lines defined edges in *Pan North XI*, in *Brass Band*, they seem more like contours. A consequence of this is that there are no points at which three distinguishable lines diverge in different directions, which means that there
are no obvious three dimensional paradigms with which to judge all the other shapes. If there are three-dimensional objects in this image, the lines transverse the transitions between their faces, rather than outlining their boundaries. Also, apart from the shape in the bottom right corner, viewers see at most two “sides.”

There is limited evidence of perspective here, too. Only in the skinny, angled shape in the right half do we see lines that might be converging in the distance. Adding to the line’s somewhat obfuscatory effect is that they are very clearly made of thick cord plastered to the panel. Their obvious two-dimensionality makes it harder to extract depth from them.

Most indicative of a third dimension in this painting are the overlapping edges of the different shapes. Even though the big arrow shape in the bottom has no shading, third face, or converging contours, the fact that it seems to be on top of the shape to its right, but under the shape on its upper left, situates it a little bit in space and suggests depth. Below some of the edges are even shadow-like dark spots that further support such an interpretation. In all, this painting in some places suggests depth and in others emphasizes its flatness, and the relationships between the direction vectors—if there is one—is very unclear.

**Background on Brooklyn Bridge**

The fracturing in *Brooklyn Bridge* adds new considerations. It distorts the painting and makes viewers piece together the fragments of the bridge, and (if they don’t know the title) consider what is actually being depicted. In this way, the painting presents a more challenging extension of the type of reasoning used in *Pan North XI*, in which viewers find relationships between different established directions and relate them to others.

By interrupting the standard process of spatial orientation, *Brooklyn Bridge* also opens the door for more abstract interpretations. While *Brass Band* might make the viewer decide whether and why it depicts as many dimensions as our normal lives, *Brooklyn Bridge* might make viewers consider if it actually illustrates more dimensions than our normal lives—and if it does, how. Students might also question if it illustrates multiple realities, or perhaps the same one at multiple times. And even if viewers decide that it doesn’t depict any extraordinary scenes, the fragmentation still isolates the effects of the directions, without distracting viewers with the vehicles of those effects. For example, one pronounced quality is the vertical thrust produced by lines that point upward and converge at the top of the painting. It is of primary concern that your attention is being drawn upwards; that it is being drawn upwards toward the top of a bridge is less important. When they practice identifying such effects of the direction vectors, students might use them to organize the painting and their responses to the painting, and, ultimately, might better understand the concepts and dynamics of the space-spanning vectors behind them.
Vectors and Space

What is a vector?

Vectors are quantities that have magnitude and direction. They are all-important in physics because they describe quantities that exist in the physical, three-dimensional world. One such quantity is displacement, which is defined as a change in something’s position. Like all vector quantities, it needs direction to have physical meaning: if you move, you have to move in some direction. Moving in no direction is equivalent to not moving at all.

How do vectors define space?

Like displacement, all vectors involve a change in some quantity, and as such are depicted as arrows connecting two points. To combine vectors, you pin the tail of the second vector to the tip of the first. The resultant vector is the one made by connecting the start of the first vector directly to the end of the second. If the vectors are parallel, this is the same as taking just one step forward instead of three forward then two back. If the vectors are perpendicular, combining them is like taking five steps at an angle, rather than three forward then four sideways.

It’s also possible to stretch or squeeze a vector. This gives you more options when trying to compose a new one. Instead of taking three steps forward and two back, for example, you could also just take 3 one-third-sized steps forward and reach the same destination. In one dimension, it is actually possible to create any vector just from stretching or shrinking any other. But the key point is that when you stretch a vector, you can’t pivot it. So in two dimensions, you need at least two non-parallel, or linearly independent, vectors to make any other. It’s like the difference between a number line and an x-y graph. On the number line, every point can be described by a distance from zero. On an x-y graph you need two distances, one in the x-direction and one in the y-direction. This is the concept of a basis: a set of the minimum number of vectors needed to compose any other vector in space. A basis generates the space. The dimension of a space is the number of vectors in one of its bases. Then in order to describe any vector in three dimensions, you need a third direction, the z-direction. And so on.

The leftmost picture shows two parallel vectors combining. Since the two of them are not linearly independent, they only generate one dimension. The middle picture shows two linearly independent vectors combining. They span two dimensions. The rightmost picture shows three linearly independent vectors combining. They generate a three-dimensional space.

How is this different from a coordinate plane?

The essence of what we’ll do is not so different from constructing coordinate graphs in painting. The most convenient bases consist of vectors that are perpendicular to each other, like
the x, y, and z-axes. But it is still possible to span a given space with skewed axes; the recipe for a given vector will just change a little if it is to be in terms of the new axes. (This exact concept is often used in special relativity, in graphs called Minkowski diagrams, to describe frames of reference of objects travelling at different speeds.)

Skewed axes are interesting to look at because they might define two dimensions in a suboptimal way, but they also happen to be the way we often depict more dimensions. In these cases, we place additional axes skew to the x and y-axes and assume that they are all perpendicular, even though they don’t seem so on paper. Ultimately, it is up to the viewer to interpret the “new” axis as depth, rather than as just a diagonal line. Perceiving a third dimension obviously comes quite naturally a lot of the time. But in more abstract works, it can be an exciting exercise to judge the dimensions at play, and to grapple with the limits of the medium and of our own capacity to perceive dimension. Adding to this exercise is the concept of perspective, which is the effect that makes lines stretching into the distance seem to converge at a distant point. Perspective also makes you consider which lines are really parallel and which are not. The activities in this lesson aim to encourage these considerations, using the vocabulary and concepts of vectors.

Finally, a point of clarification: if we consider the space in paintings, it is important to understand that physical objects themselves are not vectors. The edge of a block, for example, might be oriented along the x-direction vector, or it might make viewers think of certain vectors, but the edge is not, strictly speaking, a vector itself.

Guiding Questions

In this lesson, you will use paintings to visualize vectors and to describe space. For each painting, you might consider the following questions:

1. What do you see? Does anything seem like it might be a vector? How do you know? Is it big or small? What direction does it point?

2. If you can find any vectors, do they seem to interact with each other? How do their sizes relate? Are they parallel? Perpendicular? Somewhere in between? How many dimensions do they generate?

3. Do these relationships affect the painting as a whole? If so, how? Are they reminiscent of any other scenes or ideas?
Observation #1 – *Pan North Xi*

1. What do you see? Be specific. Do you have any immediate reactions? What are they?
2. Recall the definition of vectors. Do you see any implied here? What do they look like? Are they large or small, and in what direction do they point?
3. How do different direction vectors relate to each other? What is the dimension of the space they generate?
4. Imagine there are no colors. Are the lines able to communicate all the same relationships by themselves? Or is something different?

*Use the space below to record your thoughts.*

Observation #2 – *Brass Band*

1. What do you see? Be specific. Do you have any immediate reactions?
2. Look at the lines. What directions do they point in? Compare them to each other. Try to imagine the planes that the connected lines generate. How many dimensions do you think are being depicted here?
3. Now imagine the process of making this painting, of John Covert plastering the cord to the panel. Focus on the tangibility of the cords. Does that affect the way you perceive space in this painting? Why? How does its use, compared to paint, limit or expand possibilities for the artist?
4. What is the role of the shadows? Are they the primary source of depth, or could you imagine the lines playing the same role? If so, how?
*What is the Most?* — Explore this room in the gallery and match each of the descriptors with the painting you think fits it most. You may use the same paintings however many times you want. After 10 minutes, come back to the group and discuss.

<table>
<thead>
<tr>
<th>Flattest</th>
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<tbody>
<tr>
<td>Deepest</td>
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<tr>
<td>Highest Dimension</td>
<td></td>
</tr>
<tr>
<td>Lowest Dimension</td>
<td></td>
</tr>
<tr>
<td>Most Cluttered</td>
<td></td>
</tr>
<tr>
<td>Most Organized</td>
<td></td>
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</tbody>
</table>

**Observation #3** — (The title would give it away!)

1. What do you see? Focus on the individual elements before you try to figure out what this is a picture of. Are your eyes drawn to any points, in any directions? Why do you think that happens? Do these effects make you think of any specific places or types of places you know of?
2. What do you think this is an image of?
3. Now read the label next to the painting. The title will tell you what the image is. How is it affected by the fracturing? Do you think this object is the real version, or an imaginary version? Are there other objects depicted, besides the titular object? Maybe it’s the same object at multiple times?
4. Are these thoughts informed by conceptions of dimension? How many dimensions are there here? Any more or any less than what the real, physical object exists in?
**Drawing Activity** — Spread out and draw!

Draw a scene in the space below, using only straight lines if you can. It can be this gallery, or some other place or event you’re familiar with. You can use perspective or any other tool you want to communicate space—note that it doesn’t have to look “realistic!”

Once you’re happy with your original scene, try to add extra dimensions to it. There are no right or wrong ways to do this. Think about the techniques in the paintings we discussed, and invent your own techniques if you can!
# Symmetry and Emergence

## Lesson Overview

In this lesson, students will use selected paintings to help them recognize symmetry, and question its role in different systems. For each painting, students will consider the following questions:

1. What do you see? Do you see any symmetries? If so, what kind of symmetry is it—how would you define it?

2. Upon closer examination, are the symmetries perfect? Does it seem like symmetries are emergent from underlying chaos, or that imperfections are emergent from underlying order?

3. Are the symmetries you see consistent over time? Do you pay attention to different things over time? What about at different scales? Do any of these have more to do with the painting’s intrinsic characteristics than the others?

4. Are there any implications in choosing to call something a symmetry?

## Materials Overview

For this lesson, each student will need a writing utensil and paper. In addition to worksheets for each student, the instructor might find it helpful to carry extra sheets of blank paper if any student needs more room to write or draw.

The paintings selected for this lesson are *Trans Flux* (Kenneth Noland, 1963), *Number 4* (Jackson Pollock, 1949), and *Brooklyn Bridge* (Joseph Stella, 1919-20). The teacher or guide may select others as desired. Paintings with very obvious symmetries or abstract paintings with compositions that take up the whole canvas might make for the best examples.
## Suggested Itinerary: Total Time: 1 hour 10 minutes

<table>
<thead>
<tr>
<th>Time</th>
<th>Location</th>
<th>Activity</th>
</tr>
</thead>
</table>
| –          | YUAG Lobby                                         | ○ Welcome and Introduction  
○ Lead students to *Trans Flux.*                                             |
| 15 minutes | *Trans Flux* – Modern and Contemporary Art Collection | ○ Introduce concept of symmetry, and how we often approximate things as symmetric  
→ If students have not read background and overarching questions, give them time to do so.  
→ Introductory activity: If students say faces are symmetric, have them compare selfies with a camera app to pictures in a mirror or selfies on an app like Snapchat that doesn’t flip images.  
○ Introduce *Trans Flux* (Title, Artist, Year)  
→ Observation #1 Questions  
→ After 5 minutes, discuss student responses for another 5 minutes. |
| 25 minutes | *Number 4* – Modern and Contemporary Art Collection | ○ Introduce *Number 4* (Title, Artist, Year)  
→ Observation #2 Questions  
→ After 5 minutes, discuss student responses for another 5 minutes.  
○ “What is the Most?” activity.  
→ Tell students to spread out and to use all the paintings in the room.  
→ After 10 minutes, gather and discuss answers. |
| 30 minutes | *Brooklyn Bridge* – American Paintings and Sculpture | ○ Introduce *Brooklyn Bridge* (Title, Artist, Year)  
→ Observation #3 Questions  
→ After 10 minutes, discuss student responses for an additional 5 minutes.  
○ Drawing activity  
→ After 20 minutes, return to the group and discuss.  
○ Conclude, have students share anything they’ve learned. |
**Background on Symmetry:**

**What is symmetry?**

Symmetry is the property that preserves something after some action is done to it. Just as there are a lot of possible actions, there are different types of symmetries. For example, if something is mirrored on either side of an axis, it has what’s called a bilateral symmetry. If something is the same in different places, it has a translational symmetry. Rotations are common operations used to describe symmetries, and the rotations that preserve the original shape form the symmetry group. For a circle, this symmetry group is infinite because it looks the same at any angle of rotation. But a square, on the other hand, only stays the same if one of its corners ends up in the same place or in the place of one of the other corners. So squares only have four rotations in its symmetry group: those of 0, 90, 180, and 270 degrees.

**How is symmetry relevant to physics?**

Symmetry often simplifies problems, usually by allowing physicists to cancel out otherwise complicating terms. In a broader context, symmetry is also a sign of consistency of the laws of nature. Time and space translations are understood as sorts of axioms in physics: the laws of nature are the same—always and everywhere. But it might become problematic when “symmetry” is formally appealed to in the process of derivation, especially when the appeals call to more specific types of symmetry. Sometimes, doing so works out. Einstein reasoned that the laws of nature should be the same in all frames of reference, and relativity was born. But other times, appealing to symmetry can be criticized as overly romantic, or as mistaking convenience for providence. An example of this is supersymmetry in string theory, which—maybe-justifiably, maybe-naively—predicts the existence of several yet-undiscovered particles on the basis of symmetry with existing counterparts.

**Does symmetry have to be perfect?**

This is an interesting question because in nature, symmetry is almost never perfect. This is true in physics and in everyday perception—we often call human faces symmetric, for example, but they never actually are. Similarly, physical models often represent idealizations. This prompts a question: if we never actually see symmetry in nature, why should underlying laws necessarily be symmetric? This is also where the question of emergence arises. To what extent are natural symmetries just serendipitous arrangements of random conditions? Or is asymmetry just a result of symmetric components that are layered in such a complex way that we no longer recognize their symmetry? Asking these questions forces scientists and students to really consider the role and value of symmetry in science, and to reconsider the science they know under a different lens.

**Background on Trans Flux:**

*Trans Flux* was painted by Kenneth Noland in 1963. It has a confident sense of direction and a precise color progression, but perhaps the most immediate characteristic is that it is symmetric down the middle. If you looked at the painting in a mirror, it would look pretty much the same. For the sake of involving some math, one might say it resembles a plot of $y = |x|$. As
you move away from the middle of the painting, the action (the three lines) moves farther up. What makes it symmetric is that $y$ has the same value for positive and negative $x$. If this represented a physical system, scientists could ignore one half of it, work out the math for the other half, and effectively know how both sides behave (ignoring the boundary where both halves meet, which might be more complicated).

The symmetries are fairly limited, though. Rotating the painting clearly wouldn’t produce a symmetry unless it’s rotated a complete 360 degrees, and neither would any reflection besides the one about the central vertical axis. Even the bilateral symmetry itself is slightly limited. Looking closely, it is easy to discern imperfections in the edges of the chevron shape, as well as a couple stray drops of paint. Do these imperfections invalidate the initial symmetry? There isn’t a correct answer. Maybe viewers will feel the initial symmetry is primary because it describes the overarching composition more effectively, even if only as an approximation. Or maybe the viewer will feel the only real symmetry is perfect symmetry. But even then, the viewer must admit that squinting does make it look awfully symmetric. Maybe the viewer thinks symmetry and asymmetry both characterize the painting, and the more important question is on which scale to consider. Finally, each viewer might recognize different analogs for natural laws—the artist’s intentions of symmetry versus his actual achievement of symmetry. In each case, the underlying question is which of the overarching symmetry of underlying randomness occurs in spite of the other.

Another point worth considering is the relationship of the white space to the painted chevron space. The white space might be viewed as just the background, an underlying ether that the chevron is simply on top of. But it might also be an object in its own right, that also follows $y = |x|$. The answer to this question might affect the viewers understanding of the previous ones.

**Background on Number 4:**

Jackson Pollock painted *Number 4* in 1949. Representative of his drip technique, it is spontaneous, energetic, and complex. Some critics praise this technique as effectively communicating the artist’s subconscious, rather than constricting it into recognizable images. Unlike in *Trans Flux*, if the Pollock had any rules or symmetries in mind, it doesn’t seem he cared to make them obvious. To some viewers, the chaos of the paint drips might seem a clear counterexample to the ordered symmetry in *Trans Flux*.

But imagining several transformations on *Number 4* might test the association of symmetry, order, and disorder. If the painting were to be rotated or reflected about any axis, its original state would definitely not be preserved. But, then again, squinting might make it much harder to discern any differences. For example, noting that the thick gray drips and the black drips are perhaps the most prominent, and that they seem to be more dense towards the bottom, it might be said that a reflection about the central vertical axis (just like in *Trans Flux*) mostly preserves the most salient features. This is obviously a very shallow and incomplete assessment of this painting, but it is valid to say that, for some reason, this painting conveys a sense of uniformity—that can also be preserved under some actions—despite the fact that no two subsections are the same. This is reminiscent of the translational symmetry that supposes the laws of nature are similar everywhere. In *Number 4*, even though different manifestations of each
“law” may be activated in different places, the laws themselves are constant. This interpretation might suggest that “symmetry”, when evaluated less strictly, might be related to “uniformity”.

At the same time though, it might pay to be suspicious of an emergent symmetry such as this one. It could be that there is so much going on in *Number 4*, that seeing symmetry is a subconscious attempt to make sense of it all. While *Trans Flux* might be an imperfect symmetry, *Number 4* could be an example of the inverse, whatever that may be. Recalling the first characterizations of “spontaneous” and “energetic”, *Number 4* might also make students question whether symmetry is inherently involved with either movement or rest. Regardless of consensus, between the extremes of this painting and *Trans Flux*, we can form a scale on which to evaluate others.

**Background on Brooklyn Bridge**

*Brooklyn Bridge* is somewhere in between *Trans Flux* and *Number 4*. Like *Trans Flux*, there seems to be a degree of explicit bilateral symmetry. In the central column of the canvas, the bridge’s tower, the walkway, and the tunnel-like image and the diamond shape beneath it are all roughly symmetric in their own rights. To their sides, we find, from top to bottom, skyscrapers, cables, and tunnels. The left side isn’t a mirror-image of the right, but there is an approximate one-to-one correlation between the images that appear. Most of these pairs also share general position and orientation, suggesting loose conditions of symmetry, if not the strict symmetry seen in *Trans Flux*.

Like *Number 4*, at a smaller scale, local asymmetries take over. Differing patterns of fracture and shading dominate, and they make that area look less like its partner on the other side. But still, the apparent symmetry might seem a result more of defined patterns than it is of a sensory overload.

A unique characteristic of *Brooklyn Bridge* is that it introduces a tension between the painting and a reality that viewers know much better. From the point of view of someone on the walkway, viewers can imagine that the bridge would look very symmetric in person. The question then arises whether the painting strives to achieve this reality or to break away from it, or whether the viewer’s association with the “real” Brooklyn Bridge is valid at all.
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Guiding Questions

In this lesson, you will try to recognize symmetry in the selected paintings and use them to question symmetry’s role in different systems. For each painting, you might consider the following questions:

1. What do you see? Do you see any symmetries? If so, what kind of symmetry is it—how would you define it?

2. Upon closer examination, are the symmetries perfect? Does it seem like symmetries are emergent from underlying chaos, or that imperfections are emergent from underlying order?

3. Are the symmetries you see consistent over time? Do you pay attention to different things over time? What about at different scales? Do any of these have more to do with the painting’s intrinsic characteristics than the others?

4. Are there any implications in choosing to call something a symmetry?
Observation #1 – *Trans Flux*

1. What do you see? Be specific. Do you see any symmetries? If so, what are they? How would you define them?

2. Get a little closer to the painting and look at the edges of the chevron shapes. Are they perfectly straight? Is each imperfection the same on each side? Does this change your mind about its symmetry? If it does, try squinting. Does this do anything? What does scale have to do with symmetry?

3. What is the relationship between the white space and the chevron shapes? If you could lift up the chevrons, would it be white underneath? Why? Does your answer to this question affect any other reactions to this painting?

*Use the space below to record your thoughts.*

Observation #2 – *Number 4*

1. What do you see? Are any parts of this painting similar to others? Think of rules the artist might have followed. Would you call anything in this painting symmetric? Why or why not?

2. As you look at it longer, do any new features jump out at you? What are they? Do they help you understand the painting, or make it more confusing? Do you think your immediate reactions or your evolving reactions more accurately reflects the artist’s intention?

3. Consider this painting at different scales. Does it look any more or less symmetric if you zoom really far in or really far out? Does this tell you anything about the painting?
**What is the Most?** — Explore this room in the gallery and match each of the descriptors with the painting you think fits it most. You may use the same paintings however many times you want. After 10 minutes, come back to the group and discuss.

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<thead>
<tr>
<th>Most Symmetric</th>
<th>Least Symmetric</th>
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<tr>
<td>Most Realistic (Is Symmetry important?)</td>
<td>Least Realistic (Is Symmetry important?)</td>
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<td>Most Uniform at Different Scales</td>
<td>Least Uniform at Different Scales</td>
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**Observation #3 – Brooklyn Bridge**

1. Think about how you’ve discussed symmetry in the previous paintings. Now compare this painting to them. What’s similar? What’s different? Is it more like one than the other?
2. Does it matter that this depicts a concrete object, when the *Trans Flux* and *Number 4* are entirely abstract? Do you imagine this painting is a “realistic” image that started to fracture? A chaotic image that is converging into a recognizable image of the bridge? Neither? Why?
3. If you have an answer for the question above, how did the concept of “reality” affect it? What does reality have to do with symmetry? Anything?
**Drawing Activity** – Spread out and draw!

The goal of this exercise is to add or take away some symmetry to a painting in the gallery. In light of everything we’ve discussed, find a painting you think is interesting. Think of it on a scale from *Trans Flux* to *Number 4*. Which is it more like?

Recreate the painting you chose, but if you think it’s more like *Trans Flux*, recreate it in a way that’s more similar to *Number 4*, or vice versa. You can replicate techniques of any of the three artists we looked at, or make up your own. There are no wrong answers!
The Quantum, Continuity, and Abstraction

Lesson Overview

In this lesson, students will use selected paintings to help them evaluate abstraction and what it might communicate, and to understand some basic ideas of quantum theory. When observing the paintings, they will consider the following questions:

1. If the artist uses abstraction to simplify the composition, what are the fundamental units the artist decides are necessary to effectively communicate the scene?

2. Would you call this painting realistic? Why or why not? Is more or less information being told than is told in real life? Does anything you see make you think of any kinds of superposition?

Materials Overview

For this lesson, each student will need a writing utensil and paper. In addition to worksheets for each student, the instructor might find it helpful to carry extra sheets of blank paper if any student needs more room to write or draw. This lesson includes background reading for students and instructors to complete ideally before arriving at the YUAG.

The paintings selected for this lesson are Landscape with Water Mill (Paul Cezanne, ca. 1871), The Knife Grinder (Kasimir Malevich, 1912-13), and Brooklyn Bridge (Joseph Stella, 1919-20). However, many items in the collection might be useful, too. Paintings that have fractures, multiple images of the same thing, or other kinds of repetition might make for good examples.

Suggested Itinerary: Total Time: 1 hour 15 minutes

<table>
<thead>
<tr>
<th>Time</th>
<th>Location</th>
<th>Activity</th>
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<tbody>
<tr>
<td>–</td>
<td>YUAG Lobby</td>
<td>○ Welcome and Introduction&lt;br&gt;○ Lead group to European Art collection</td>
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<tr>
<td>15 minutes</td>
<td>Landscape with Water Mill</td>
<td>○ Review background reading, focusing on the ideas of fundamental quanta and superposition.&lt;br&gt;→ If students have not read the background or guiding questions, give them time to do so.</td>
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<td>Duration</td>
<td>Collection</td>
<td>Activity Details</td>
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<tr>
<td>10 minutes</td>
<td>European Art Collection</td>
<td>- Introduce <em>Landscape with Water Mill</em> (Title, Artist, Year)</td>
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<td>→ Observation #1 Questions</td>
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<td>→ After 5 minutes, discuss student responses for another 5 minutes.</td>
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<tr>
<td>10 minutes</td>
<td><em>The Knife Grinder</em> – Modern and Contemporary</td>
<td>- Introduce <em>The Knife Grinder</em> (Title, Artist, Year)</td>
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<td>Art Collection</td>
<td>→ Observation #2 Questions</td>
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<td>→ After 5 minutes, discuss student responses for another 5 minutes.</td>
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<tr>
<td>20 minutes</td>
<td><em>Brooklyn Bridge</em> – American Painting and</td>
<td>- Introduce <em>Brooklyn Bridge</em> (Title, Artist, Year)</td>
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<td>Sculpture Collection</td>
<td>→ Observation #3 Questions</td>
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<td>→ After 5 minutes, discuss student responses for an additional 5 minutes.</td>
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<td>- “What is the Most?” activity.</td>
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<td>→ Tell students to spread out and to use all the paintings in the room.</td>
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<td>→ After 10 minutes, gather and discuss answers.</td>
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<tr>
<td>30 minutes</td>
<td>American Painting and Sculpture/Modern and</td>
<td>- Drawing activity</td>
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<td></td>
<td>Contemporary Art Collection</td>
<td>→ After 20 minutes, return to the group and discuss.</td>
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<td>- Conclude, have students share main takeaways.</td>
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**Background on The Development of Quantum Theory**

**How did it start?**

Quantum Theory was born in 1900, when Max Planck (who was at the time, one of the world’s most famous physicists) was commissioned by an engineering company to study blackbody radiation. Blackbody radiation refers to the electromagnetic (i.e. light) waves released by a “blackbody”, an idealized object that perfectly absorbs all the electromagnetic waves incident on it. Previously, the energy emitted by such things was understood to be directly and continuously proportional to the frequency of light shone on it, but theory had never been able to explain experimental data. Planck was able to do so, but only by denying the continuity of the energy spectrum. He discovered that the amount of energy was actually constrained to multiples of a discrete fundamental amount, called an energy quantum. At larger scales, they blend together and are indistinguishable, and only when you look very, very closely do they matter.
The decision to utilize energy quanta was initially seen as just a mathematical trick to make the model work, until Albert Einstein expanded on it in 1905. Einstein showed that the quanta were not just characteristic of the energy of blackbody radiation, but characteristic of light itself. Doing so conclusively reconciled the particle-like properties and wave-like properties that had confused scientists for a couple hundred years, and it was actually for this discovery, rather than relativity, that Einstein won the 1921 Nobel Prize in Physics.

What is Quantum Mechanics?

Einstein’s theory then proved useful to Niels Bohr’s study of the atomic nucleus, and ultimately inspired the 1924 doctoral thesis of Louis de Broglie. In it, he theorized that not only do light waves exhibit properties of particles, but normal particles exhibit properties of waves. Finally, in response, Schrödinger and Heisenberg developed their equivalent wave mechanics and matrix mechanics, respectively.

It might be a little unclear what is meant by particles behaving like waves. It means that the way we see things, as having concrete location and energy, is not the whole picture. Observable characteristics are actually defined by probability waves, which specify not only where and how the object probably is, but also every place and every way and it possibly could be. This ultimate wave is a composition of all the individual possible states. It is a sort of weighted average of states, where the weights indicate a state’s relative probability. Probability implies uncertainty, and this is the foundation of Heisenberg’s famous uncertainty principle, which says that knowing information about a particle’s location comes at the cost of information about its momentum.

Another important concept is the wave function collapse. The reason objects around you aren’t hopping all over the place between all their existing possible states is that the act of measuring something makes the probability function collapse around a much narrower, almost singular state. Let’s call it “State A”. It is also important to understand that before the collapse, the object was not already in State A, and we just didn’t know it yet. Actually, before the measurement the object was, mathematically, in every one of its possible states, just to different extents. This is why “curiosity kills the cat.” Before opening the box, the poison’s atomic trigger is both switched on and off. The cat’s fate only collapses to alive or dead when we get curious and open the box.

So what?

Quantum Theory shook up the world of science and philosophy. Reality and causation were no longer as definite as they once were. The premise of this lesson plan is that 1) It might be fun and thought-provoking to examine progressive abstraction in light of quantum mechanics and 2) Without dwelling on the precise nature of the relationship, the transition from classical realistic representation to progressive abstraction and fracturing might serve as an aesthetic and thematic parallel for the development of Quantum Theory, and as a result, these paintings might help students understand how a social system of aesthetics might develop.

Background on Landscape with Water Mill:

Paul Cezanne painted Landscape with Water Mill around 1871. Cezanne is often credited as an inspiration for abstract art, and cubism especially, for the way he broke up
compositions into simple discrete shapes. In this painting, he does this with a Water Mill on a hillside, and allows his loose brushstrokes to portray a somewhat unraveled version of the original scene.

A question we might ask is how this painting reflects reality. Clearly the painting doesn’t look like a photograph of this landscape, but it might be up to the viewer to decide how much information was actually lost in abstracting it. Maybe Cezanne hoped to distill the raw image into only the shapes that are necessary to communicate the image. In this case, it is interesting to consider what kinds of shapes Cezanne decided were fundamental units.

In this painting, as objects move more into the foreground, their brushstrokes become more visible. The rocks and the mill are slightly decomposed, the trees on the hills less so, and the sky least of all. For this reason, we might also ask how scale, or level of inquiry, affects our conception of reality.

**Background on The Knife Grinder:**

*The Knife Grinder* was painted by Kasimir Malevich in 1912-13. It is so fractured and has such high color contrast that without the title as a hint, it might be very hard to see what’s going on. Let that be a starting point for students. Since Cezanne, a style has emerged that breaks the image down into even more pieces. The colors have changed, too, and are mostly pure reds, greens, blues, or grays, as opposed to the true-to-life earth tones in *Landscape with Water Mill*. From these, we can question the implications of these shifts. Why might Malevich have abstracted so much further? In what ways is it a response to the kind of abstraction Cezanne uses? Does it represent reality in a meaningfully different way?

This painting also abstracts the temporality of the base image. Or, more simply put, it adds time. The figure in the painting has extra features, indicating the different positions the knife grinder might take, and—when understood as a sequence—conveying a sense of motion. By choosing to abstract in this way, Malevich sacrifices a very precise picture of a single event for a more broad picture that might depict multiple. This tradeoff is somewhat reminiscent of the uncertainty principle. Students might wonder whether and how the constraints of *knowing* in quantum mechanics are related to the constraints of *showing* in painting.

**Background on Brooklyn Bridge:**

*Brooklyn Bridge*, like *The Knife Grinder*, is much more broken up than *Landscape with Water Mill*. But it is fractured in different ways. There are fewer bright colors and areas of high contrast in *Brooklyn Bridge*, so it is harder to distinguish the different geometries. Second, in addition to depicting the main subject (the bridge) from multiple perspectives (hence the crisscrossing cables), Stella also introduces other figures, such as the tunnels and the crystal shape at the bottom. In doing this, he seems to point out that it is impossible to experience the Brooklyn Bridge without thinking of other marvels of the American industrial landscape. And if this is the case—that what we ultimately perceive is the product of multiple places, people, and memories superimposed on top of each other—why do we say that reality isn’t all of these.

Drawing a connection to quantum theory, we might consider this a kind of visual representation of a wave function. Instead of asking what *is* present, we might ask which reality is just the *most* present.
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**Guiding Questions**

In this lesson, you will evaluate abstraction in selected paintings, think about what that abstraction might mean, and practice discussing some basic ideas of quantum theory. When observing the paintings, you might consider the following questions:

1. If the artist uses abstraction to simplify the composition, what are the fundamental units the artist decides are necessary to effectively communicate the scene?

2. Would you call this painting realistic? Why or why not? Is more or less information being told than is told in real life? Does anything you see make you think of any kinds of superposition?
Observation #1 – *Landscape with Water Mill*

1. What do you see? Be specific. Think about the lines and shapes Cezanne uses. Does he simplify anything? If so, how?
2. Would you call this painting realistic? Why or why not? What would make it more realistic? Less?
3. Remember the background reading and quantum scale. Notice that the background is less abstracted than the foreground. In this painting, what does scale have to do with abstraction?

*Use the space below to record your thoughts.*

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Observation #2 – *The Knife Grinder*

1. What do you see? What’s this even a painting of? How does it compare to the previous painting?
2. Does Malevich’s additional abstraction represent reality in a meaningfully different way? What information does it communicate less effectively? More effectively? Why?
3. Notice the superposition of the figure’s different positions. How do you interpret it? Why? What is sacrificed for including these? Do the constraints of *knowing* in quantum mechanics seem to be similar to the constraints of *showing* in painting? If so, how?
Observation #3 – *Brooklyn Bridge*

1. What do you see here? How does *Brooklyn Bridge* differ from the previous two paintings, and especially from *The Knife Grinder*?
2. What is the main subject of this painting? How do you know? Are there other subjects? If so, describe them. Do they seem unrelated? Do they all take place at the same time or at different times?
3. Think about your answers to the previous question. Which part of this painting is “real”? Do you think this painting depicts one reality or multiple? None? Why? Is there any uncertainty?

*What is the Most?* – Explore this room in the gallery and match each of the descriptors with the painting you think fits it most. You may use the same paintings however many times you want. After 10 minutes, come back to the group and discuss.

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<thead>
<tr>
<th>Most Realistic</th>
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<tr>
<td>Most Abstract</td>
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<td>Most Definite</td>
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<tr>
<td>Most Uncertain</td>
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</table>
Drawing Activity – Spread out and draw!

The goal of this exercise is to add some abstraction to a painting in the gallery. In light of everything we’ve discussed, find a painting you think is interesting. Think about images or scenes that might be related to it, but aren’t shown. This could be motion, like in *The Knife Grinder*, or additional objects, like in *Brooklyn Bridge*.

The challenge is to recreate the painting you chose, incorporating the related-but-unseen images that you thought of. You can base it on techniques of any of the artists we looked at, or make up your own. There are no wrong answers!
References

**Suggested Images for Lessons**, in the order they appear above (*Brooklyn Bridge* appears in each module, but it is only shown once below):

Unless otherwise specified, all information and images are retrieved on May 1, 2020 from the Yale University Art Gallery website, at [https://artgallery.yale.edu/collection/search](https://artgallery.yale.edu/collection/search).

*Pan North XI – Al Held, 1987*
References

*Brass Band* – John Covert, 1919
References

*Brooklyn Bridge* – Joseph Stella, 1919-1920
References

*Trans Flux* – Kenneth Noland, 1963

References

*Number 4* – Jackson Pollock, 1949
References

*Landscape with Water Mill* – Paul Cézanne, ca. 1871
References

*The Knife Grinder* – Kasimir Malevich, 1912-13
Bibliography


https://vtechworks.lib.vt.edu/bitstream/handle/10919/83073/OverviewHispanicsSTEM.pdf?sequence=1&isAllowd=y.

DeVries, R. and Sales, C. *Ramps & Pathways: A Constructivist Approach to Physics with Young Children*, (Washington, D.C., NAEYC, 2011);


